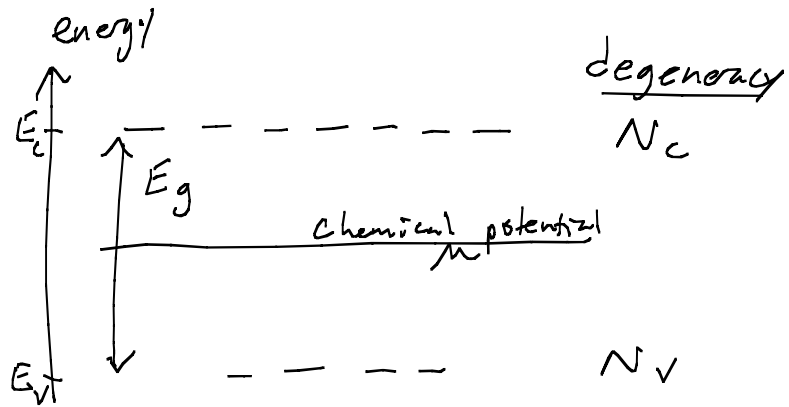


Generalized two-state system



States at $E = E_c$

$$f_c = \frac{1}{1 + e^{(E_c - \mu)/\tau}} \quad (\tau \equiv k_B T)$$

number of occupied states:

$$n_c = \frac{N_c}{1 + e^{(E_c - \mu)/\tau}}$$

For μ in gap, $E_c - \mu \gg \tau$,

$$n_c \approx N_c e^{-(\mu - E_c)/\tau}$$

→ Boltzmann distribution!

States at $E = E_v$

$$f_v = \frac{1}{1 + e^{(E_v - \mu)/\tau}}$$

number of occupied states:

$$n_v = \frac{N_v}{1 + e^{(E_v - \mu)/\tau}}$$

for $\mu - E_v \gg \tau$, expand to 1st order

$$n_v \approx N_v (1 - e^{-(\mu - E_v)/\tau})$$

define "un-occupation"

$$p_v = N_v - n_v \approx N_v e^{-(\mu - E_v)/\tau}$$

"Law of Mass Action"

$$N_c \rightarrow n$$

$$N_v \rightarrow p$$

$$\begin{aligned} np &= N_c e^{(\mu - E_c)/\tau} N_v e^{(E_v - \mu)/\tau} = N_c N_v e^{-(E_c - E_v)/\tau} \\ &= N_c N_v e^{-E_g/\tau} \end{aligned}$$

Independent of chemical potential μ !
(assuming it lies $\gg \tau$ from states in gap)

"Intrinsic" system has completely filled states @ $E = E_v$, empty @ $E = E_c$ ($T=0$). Then, only way to occupy state @ $E = E_c$ is to leave behind an un-occupied state @ $E = E_v$. Then, $n = p = n_i$; so we have

$$\underline{np = n_i^2} \quad (\text{for Si @ } T = RT = 300\text{K}, n_i \sim 10^{10} \text{ cm}^{-3})$$

"Intrinsic" System

As discussed, in this case we have

$$n = p$$

Then,

$$N_c e^{(\mu - E_c)/\tau} = N_v e^{(E_v - \mu)/\tau}$$

$$\frac{e^{(\mu - E_c)/\tau}}{e^{(E_v - \mu)/\tau}} = \frac{N_v}{N_c}$$

$$2\mu - (E_c + E_v) = \tau \ln \frac{N_v}{N_c}$$

$$\mu = \frac{E_c + E_v}{2} + \frac{\tau}{2} \ln \frac{N_v}{N_c}$$

middle of gap

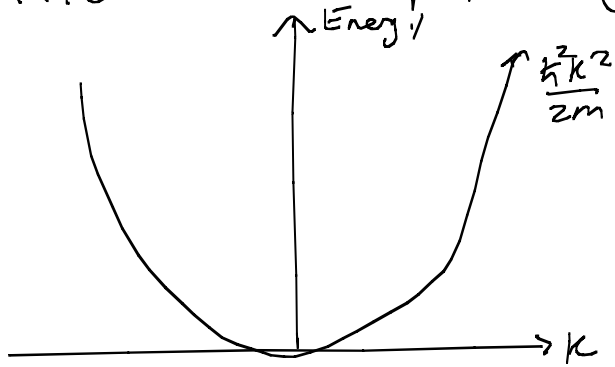
So μ gets closer to states w/ smaller degeneracy as T rises!

A real-life example of this system: Semiconductors

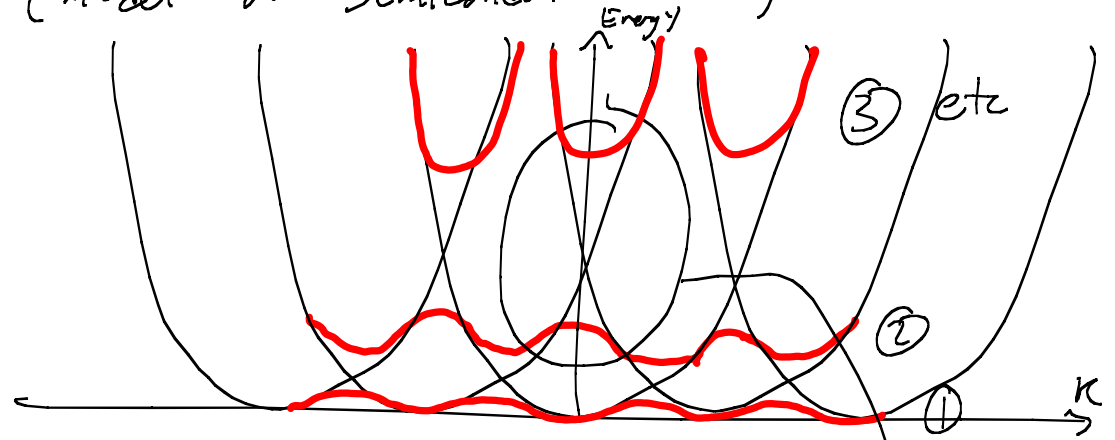
Schrodinger Eq:

$$\left[-\frac{\hbar^2}{2m} \nabla^2 + V(x) \right] \psi = E \psi$$

Free electron ($V(x)=0$):
plane-wave dispersion $\psi = e^{i(kx - \omega t)}$



Electron in periodic potential $V(x+a) = V(x)$:
(model of semiconductor lattice)



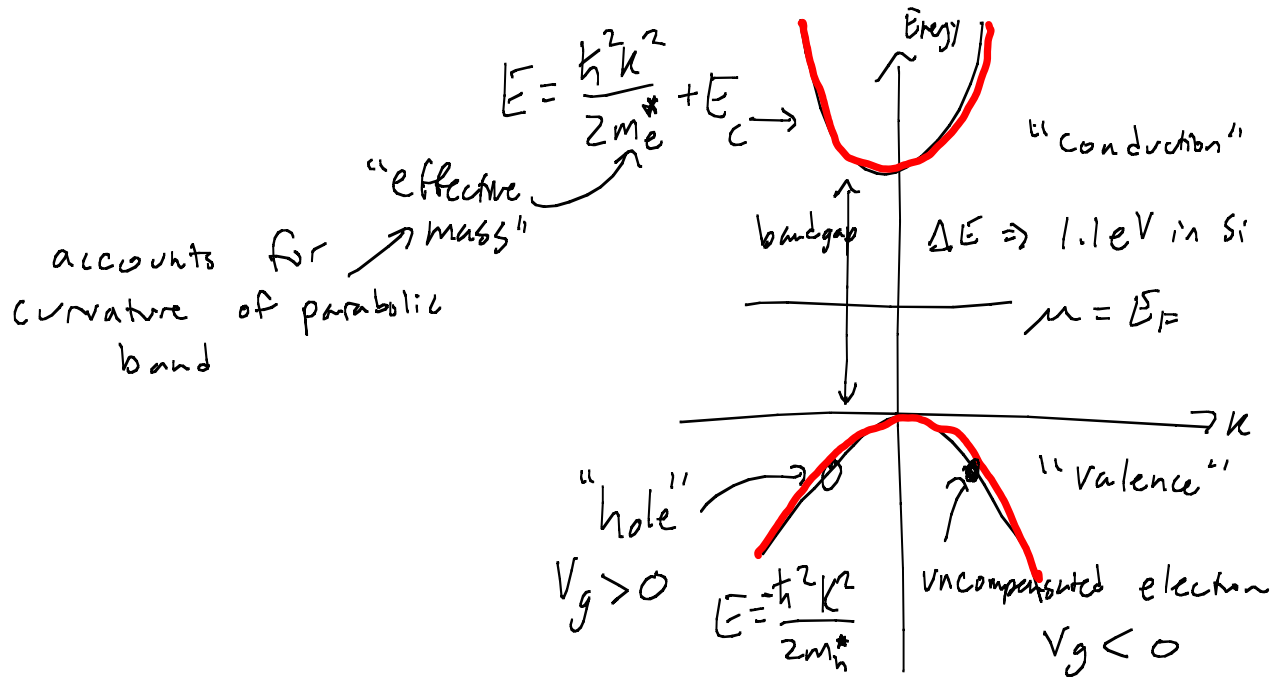
destructive interference of electron matter waves (ψ)

opens "bandgaps" between "bands" and forms "band-structure".

See detail next page

Electrons + holes

In an intrinsic semiconductor @ $T=0$, all bands are completely filled ("valence") or completely empty ("conduction")



— filled bands contribute nothing to conduction of electrons since qV_g is compensated.

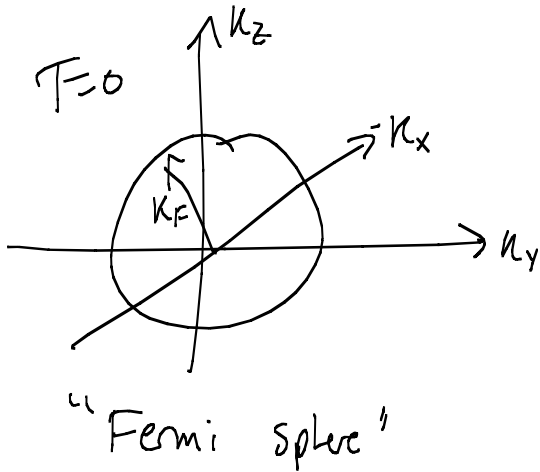
— Since $J = n \cdot (qV_g)$, we can think of an unoccupied state in valence band as a particle w/ positive charge and opposite velocity V_g as the uncompensated electron: a "hole"!

Electron concentration

$$n = \int D_{3d}(E) f(E) dE$$

↑
density of states

In 3-d @ $T=0$



$$n = 2 \iiint_{|k| < k_F} \frac{dk_x dk_y dk_z}{(2\pi)^3} \xrightarrow{\text{Spherical Symmetry}} 2 \int_0^{k_F} \frac{4\pi k^2 dk}{(2\pi)^3}$$

↑
sph

change of variables $E = \frac{\hbar^2 k^2}{2m_e^*}$

$$k = \frac{\sqrt{2m_e^* E}}{\hbar}, \quad dk = \sqrt{\frac{m_e^*}{2}} \frac{E^{-1/2}}{\hbar} dE$$

$$= 2 \int_0^{E_F} 4\pi \frac{2m_e^* E}{\hbar^2} \sqrt{\frac{m_e^*}{2}} \frac{E^{-1/2}}{\hbar} \frac{dE}{(2\pi)^3}$$

$$= \int_0^{E_F} \underbrace{\frac{2^{1/2} m_e^{*3/2}}{\pi^2 \hbar^2} E^{1/2}}_{D_{3d}(E)} dE$$

$$S_0 \quad n = \int_{\text{CBM}}^{\infty} \frac{2^{1/2} m_e^*{}^{3/2}}{\pi^2 \hbar^2} E^{1/2} e^{-(E-E_F)/\tau} dE = \left(\frac{2^{1/2} m_e^*{}^{3/2}}{\pi^2 \hbar^2} \right) e^{E_F/\tau} \int_{\text{CBM}}^{\infty} E^{1/2} e^{-E/\tau} dE$$

change of variables to $x \equiv E/\tau$ $E = x\tau$, $dE = \tau dx$

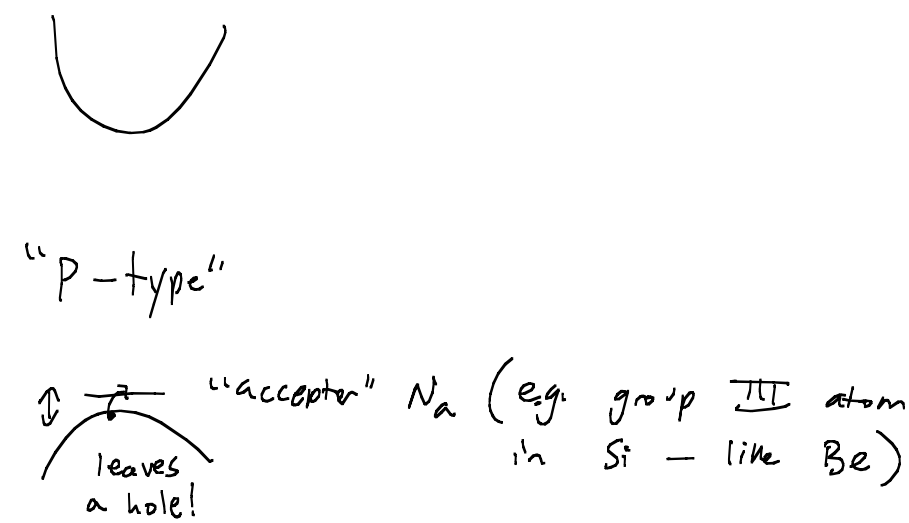
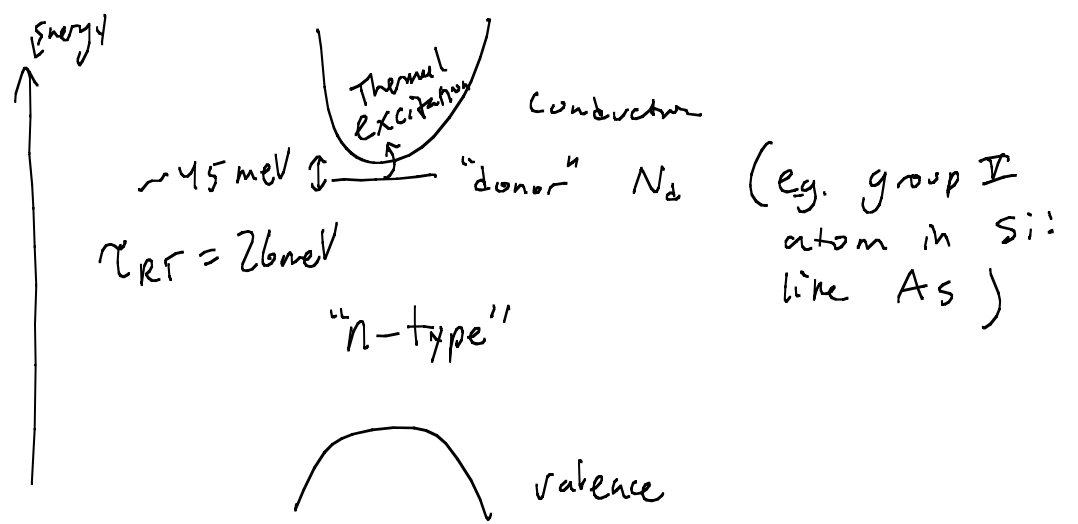
$$= \left(\frac{2^{1/2} m_e^*{}^{3/2}}{\pi^2 \hbar^2} \right) e^{E_F/\tau} \tau^{3/2} \underbrace{\int_0^{\infty} x^{1/2} e^{-x} dx}_{\frac{\sqrt{\pi}}{2}}$$

CBM is at $E = E_c$:

$$= 2 \left(\frac{\tau m_e^*}{2k^2 \pi} \right)^{3/2} e^{(E_F - E_c)/\tau} = \underbrace{N_c}_{\text{effective conduction band Dos}} e^{E_F - E_c/\tau}$$

So a semiconductor is equivalent to the two-state system on p1!

"Extrinsic" Semiconductor: "doping" to control E_F



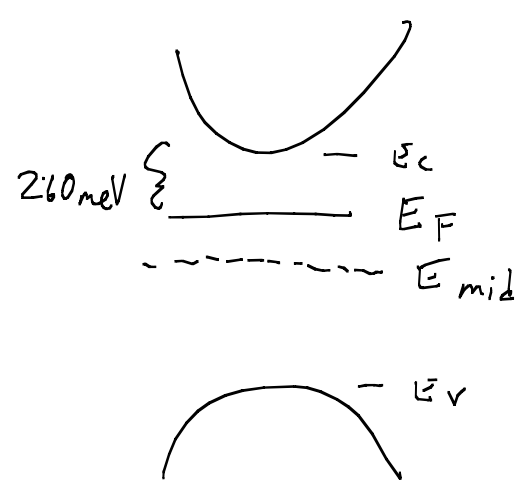
Example:

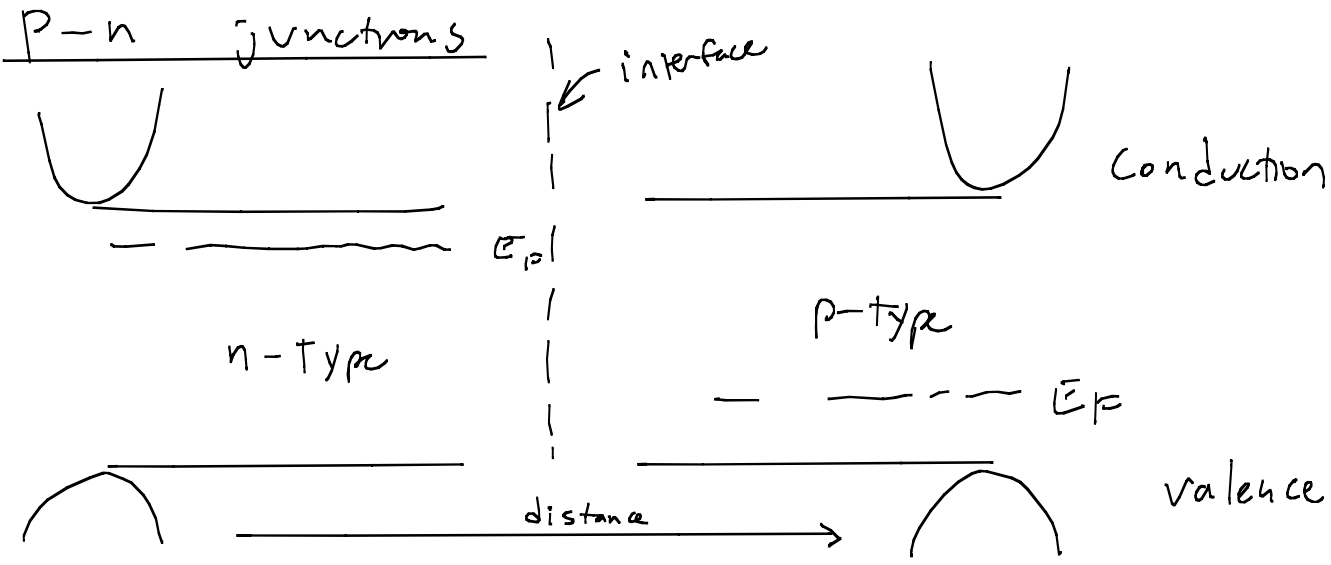
$N_d = 10^{15} \text{ cm}^{-3} \rightarrow$ where does E_F lie?

$$n \approx N_d = N_c e^{(E_F - E_c) / k_B T}$$

$$E_F - E_c = k_B T \log \frac{N_d}{N_c} = 0.026 \text{ eV} \cdot \log \frac{10^{15}}{2 \times 10^{19}} = -0.26 \text{ eV}$$

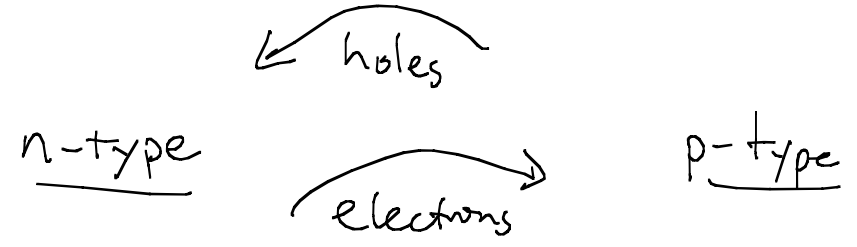
$2 \times 10^{19} \text{ cm}^{-3}$ for Si @ RT



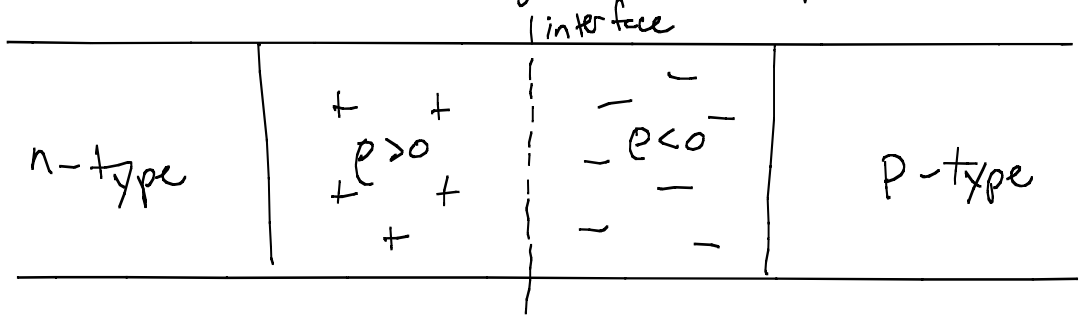


plots of conduction band minimum (CBM) and valence band maximum vs. distance are called "band diagrams"

but in equilibrium, E_F must be equal! So we get charge transfer

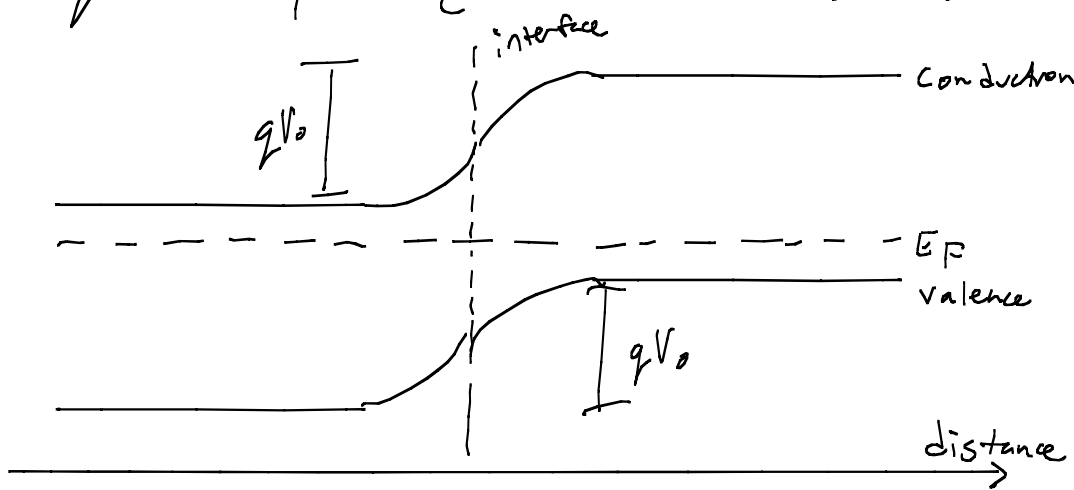


leaving behind depletion regions exposed charged ions from dopants:



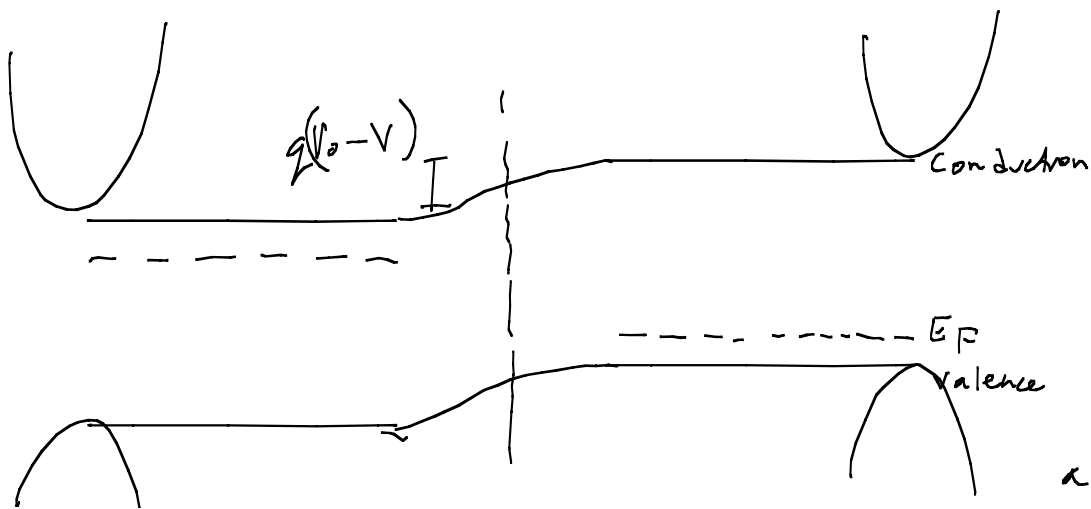
Electrostatics

Poisson's eq: $-\nabla^2 \phi = \frac{\rho}{\epsilon}$ \rightarrow Solns are quantized in x :



w/ built-in potential barrier
 qV_0 keeps electrons on n-side + holes on p-side.

Under bias w/ voltage V applied to p-type side, The bands look like:

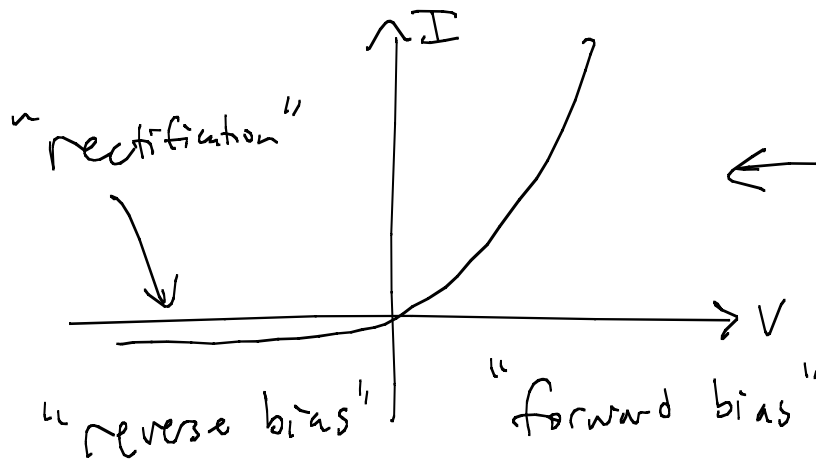


The applied voltage can change the barrier height. So that under positive bias, the barrier is lowered, allowing energy states w/ relatively large occupation to couple across the interface w/ unfilled states!

Ideal diode eqn

Lowering the barrier by qV increases the concentration of charge carriers by a factor of $e^{qV/k_B T}$. Since diffusion is transport driven by concentration difference, the current caused by electron "injection" into p-type side and hole "injection" into n-type side is then proportional to:

$$I \propto (e^{qV/k_B T} - 1)$$



This asymmetry is the basis of device design: npn/pnp bipolar transistors, MOSFETs, Thyristors, etc!